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*p*ISSN: 1220-871X *e*ISSN: 1844-0401

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THE TECHNOLOGICAL OPTIMIZATION OF DISCONTINUOUS FILTRATION USING GEOMETRICAL PROGRAMMING

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abstract: The paper depicts both a technological optimization method for discontinuous filtration that is based on experimental data and a computer program that realises the optimization. The algorithm and the program allow getting the optimum structure of a filtration cycle for any filter with discontinuous work. Both the modality of optimization and the computer program can be used successfully in industry for increasing the economical efficiency in the exploitation of filtering equipment.

key words: filtration; optimization; computer program

received: March 7, 2011 *accepted*: May 9, 2011

1. Theoretical Aspects

The principal characteristic of the filtration is the *filtration rate*, which represents the volume of filtrate that passes through the surface's unit in a time unit ($m^3/m^2 \cdot h$). It depends on the pressure difference, temperature, on the nature, character and thickness of the filter cake, on the composition of the suspension etc.

A filtration cycle consists of the following stages:

- *the proper filtration;*
- *the precipitate washing;*
- *the regeneration of the filtration medium* (the cake removing and the regeneration of the medium).

The duration of a filtration cycle is:

$$\tau = \tau_f + \tau_s + \tau_r \tag{1}$$

where: τ_f is the filtration time;

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τ_s – the washing time;

τ_r – the regeneration of filtration medium.

The filtration can be performed at a constant pressure difference (when the filtration rate is maximum at the beginning of the filtration), at a constant filtration rate (the pressure difference is maximum at the end of the filtration) and in the mixed mode (initial at constant filtration rate an then at constant pressure difference).

In the case of filtration at constant pressure difference, the operation takes place in non-stationary conditions, the filtration rate decreasing owing to the growth of the hydrodynamic resistance of the cake.

In this case, the filtration rate, and the filtration time, τ, are related as [2]:

$$V^2 + 2 \cdot C \cdot V = K \cdot \tau \quad (2)$$

where: C – the constant of the filtration medium, which characterises the its hydrodynamic resistance (m^2/m^3);

K – the constant of the precipitate, which takes into account the regime of filtration process and the physico-chemical properties of the precipitate and the filtrate.

τ – filtration time (s).

The constants C and K can be analytically determined [3], but their determination from experimental data is preferable.

The geometrical programming [4] is an optimization technique which is applied to a certain type of goal function, especially those having the following form:

$$y = \sum_{j=1}^{T} c_j \cdot P_j(\bar{x}) \quad (3)$$

where: c_j is a numerical positive coefficient;

$P_j(\bar{x})$ is given by the relation:

$$P_j(\bar{x}) = \prod_{k=1}^{n} x_k^{a_{kj}} \quad (4)$$

where: n is the number of independent variables;

a – an exponent; its first index, k, indicates the independent variable which it refers to, and the second, the j term which intervenes in (j = 1... T).

The polynomial sum of type (4) is named generalised polinom or *pozinom*. The c_j coefficients are, usually, of economics origin and the a_{kj} exponents, of technological origin. The general way of geometrical programming is inverse to those practised in other optimization methods, because, first, it is determined the optimal value of the optimization criterion and then the optimal solution. For calculating the optimal value of the optimization criterion, the following relation is used:

$$y^* = \prod_j \left(\frac{c_j}{w_j} \right)^{w_j} \tag{5}$$

where w_j are optimal weights of c_j coefficients [4].

Using both the knowledge about discontinuous filtration and that related to geometrical programming, one can optimize any discontinuous filter, used in any economical domain, doing his own experiments. The results of the optimizing process are strictly available on his filter and only for the same materials used in the operation and at the same temperature of the suspension. Thus, this optimization can be understand as the best solution in given conditions, but it can not be extended for other filter or other materials or other temperature used in the process.

2. Experimental Part

For exemplifying the modality of optimization of discontinuous filtration we used a laboratory filter with an area of 0.1 m^2, which needs a time for auxiliary operations at the filtration medium regeneration of 24 minutes; a wastewater containing 30 % suspensions was filtered. Experimentally, the washing time and the regeneration of filtration medium time represent about 40 % and 10 %, respectively, from the filtration time (including the time necessary for auxiliary operations). Supposing that last year 10 tons of dross have been removed using an unoptimized filtration cycle, we propose the optimization of the filtration cycle for the same quantity of dross. Starting from this example, the optimization of discontinuous filtration for an industrial filter used for the purification of wastewaters in a wastewaters purification station is possible.

As a suspension, we used a wastewater taken from the entrance of the wastewater purification station of *C. S. Antibiotice SA*, containing as impurities both industrial dross and agrarian-zoo technical dross coming from a complex of pigs' growing from the area. Having as an aim the determination of the suspension's concentration, the following operations have been done:

- wastewater sample weighing;
- dross sedimentation;
- water decantation;
- dross drying at 105°C, for two hours;
- dry dross weighing;
- calculation of the suspension concentration.

Having as an aim the determination of filtration constants from experimental data, the following experimental operations have been performed:

- the suspension homogenization in a manual mixer;
- the wastewater filtration;

– the cumulative measure of times for each litre of obtained filtrate;
– the cleaning of filter texture;
– the exhaustion of filtrate bowl;
– the aeration of filtrate bowl.

The experimental data for the analysed filter are given in Table 1.

Table 1 Experimental data for the analysed filter.

No. det.	Filtrate volume from the beginning of the filtration, dm^3	Time from the beginning of the filtration, s	V_i, m^3/m^2	$\frac{\Delta\tau}{\Delta V}$, $\frac{h \cdot m^2}{m^3}$
1.	1	16	0.01	0.444
2.	2	48	0.02	0.889
3.	3	85	0.03	1.020
4.	4	147	0.04	1.722
5.	5	216	0.05	1.917
6.	6	300	0.06	2.333
7.	7	393	0.07	2.583
8.	8	495	0.08	2.833
9.	9	610	0.09	3.194
10.	10	740	0.10	3.611

3. The Application of the Geometrical Programming for the Optimization of Discontinuous Filtration

For the optimization of the process, it is necessary to know the constants of the filtration from the equation (2). They can be found out using the graphic method, starting from the finite differences filtration's equation [3]:

$$\frac{\Delta\tau}{\Delta V} = \frac{2}{K} \cdot V + \frac{2C}{K}; \left[\frac{\mathrm{h} \cdot \mathrm{m}^2}{m^3}\right] \tag{6}$$

where: Δτ – time interval, in hours, that is necessary for the filtration of the volume ΔV, which passes through 1 m^2 of filtration surface;

V – specific volume of filtrate (m^3/m^2);

K – the constant of the precipitate (m^2/h);

C – the constant of the filtration medium (m^3/m^2).

From Table 1 one notices the increasing of time interval necessary for the same suspension volume, once the cake is accumulated, that proves the non-stationary conditions of constant pressure difference filtration.

More precious than the graphic method for the determination of the filtration constants is the least squares method [5]. The relation (6) can be approximated with a linear regressive equation:

$$\tilde{y} = b_0 + b_1 \cdot x \tag{7}$$

where:

$$\tilde{y} = \frac{\Delta\tau}{\Delta V};\ b_0 = \frac{2C}{K};\ b_1 = \frac{2}{K} \tag{8}$$

Using the differential filtration equation one can get the integrative form of it:

$$\tau = \frac{1}{K} \cdot V^2 + \frac{2C}{K} \cdot V \tag{9}$$

with:

$$V = \frac{V_t}{n} \tag{10}$$

where: V_t is the total specific volume of filtrate, in m^3/m^2;

n – the number of cycles for discontinuous filtration.

One can get the filtration time for a single cycle:

$$\tau = \frac{1}{K}\left(\frac{V_t^2}{n}\right)^2 + \frac{2C}{K} \cdot \frac{V_t}{n} \tag{11}$$

The total proper filtration time is got using the expression:

$$\tau_f = n \cdot \tau = \frac{V_t^2}{K} \cdot n^{-1} + \frac{2CV_t}{K} \tag{12}$$

The total precipitate washing time, proportional with the proper filtration time is:

$$\tau_s = \beta \cdot \tau_f \tag{13}$$

The total time for the filtration medium regeneration is:

$$\tau_r = \gamma \cdot \tau_f + n \cdot \tau_a \tag{14}$$

where: γ is a proportionality coefficient;

τ_a – is auxiliary operations of regeneration for a cycle time.

It results that the total time of the filtration is:

$$\tau_t = \frac{V_t^2 \cdot (1+\beta+\gamma)}{K} \cdot n^{-1} + \tau_a \cdot n + \frac{2CV_t \cdot (1+\beta+\gamma)}{K} \tag{15}$$

Using the notations:

$$A = \frac{V_t^2 \cdot (1+\beta+\gamma)}{K} \tag{16}$$

$$B = \frac{2CV_t \cdot (1 + \beta + \gamma)}{K} \tag{17}$$

the equation (15) becomes:

$$\tau_t = A \cdot n^{-1} + \tau_a \cdot n + B \tag{18}$$

which can, finally, be written in the form:

$$z = \tau_t - B = A \cdot n^{-1} + \tau_a \cdot n \tag{19}$$

The equation (19) is a pozinom of form (4); in line with those previously depicted, the coefficients a_{kj}, from equation (4) have the values: $a_{11} = -1$; $a_{12} = 1$.

The optimum weights w_1 and w_2 are obtained from the orthogonality and normality conditions [4]:

$$\begin{cases} w_1 + w_2 = 1 \\ -w_1 + w_2 = 0 \end{cases} \tag{20}$$

which form an equations system that, being solved, leads to the solutions:

$$w_1^* = w_2^* = w^* = 0,5 \tag{21}$$

In line with the equation (5) it results:

$$z^* = \tau_t^* - B = \left(\frac{A}{0,5}\right)^{0,5} \cdot \left(\frac{B}{0,5}\right)^{0,5} \tag{22}$$

Proceeding from the geometrical programming method [4], it results the possibility of calculation for the optimum filtration cycles number, because:

$$w \cdot z^* = \tau_a \cdot n^* \tag{23}$$

Replacing n^* in equation (18) one can get the total optimum time for a filtration cycle, and also the filtration cycle time in optimum conditions:

$$\tau_c^* = \frac{\tau_t}{n^*} \tag{24}$$

Knowing the optimal time for a filtration cycle, one can elaborate the optimal structure for a cycle of discontinuous filtration.

4. The Presentation of the Principal Modules of the Program which Realises the Optimization

Based on the algorithm previously depicted, programs in FORTRAN and TURBO-PASCAL programming languages were elaborated. The principal modules of the TURBO-PASCAL program which realises the optimization of discontinuous filtration using geometrical programming are:

```
procedure coefficients (xd,yd:vectx;
var a0,a1:real);
 var xp:matx;sum,sum1:real;
 begin
 for i:=1 to 2 do for j:=1 to 2 do
 xp[i,j]:=0;
 xp[1,1]:=n;
 for i:=1 to n do begin
 xp[1,2]:=xp[1,2]+xd[i];
 xp[2,2]:=xp[2,2]+putp(xd[i],2);
 end;
 xp[2,1]:=xp[1,2];
 sum:=0;
 sum1:=0;
 for i:=1 to n do
 begin
 sum:=sum+yd[i];
 sum1:=sum1+xd[i]*yd[i];
 end;
 xp[1,3]:=sum;
 xp[2,3]:=sum1;
 succel(2,xp,sol);
 a0:=sol[1]; a1:=sol[2];
 end;
```

```
'o':BEGIN
 clrscr;writeln(' THE CONSTANTS OF FILTRATION:');
 coefficients (v,t,b0,b1);kf:=2/b1;cf:=b0/b1;
 writeln('K=',kf,'m.p./h');writeln('C=',cf:9:5,'mc/mp');
 vf:=(100-cs)*pa*1000/(cs*ro);
 writeln('Volume of filtrate:',vf:9:5,' m.c.');vf:=vf/af;
 a:=putp(vf,2)*(1+beta+gamma)/kf; writeln('A=',a:9:5);
 b:=2*cf*vf*(1+beta+gamma)/kf; writeln('B=',b:9:5);
 zop:=putp(a/0.5,0.5);zop:=zop*putp(ta/0.5,0.5);
 writeln('zop=',zop:10:3);nop:=0.5*zop/ta;
 writeln(' Optimal number of cycles:',nop:9:0); tt:=zop+b;
 writeln('Total optimal filtration time',tt:8:2,' hours');writeln;
 writeln(' OPTIMAL STRUCTURE OF THE CYCLE:');
 tt:=tt/nop;writeln;tf:=(2*cf*vf/kf+putp(vf,2)/(nop*kf))/nop;
 writeln('Proper filtration:',tf:16:3,' hours (',round(tf*60):3,
 ' min.)');
 ts:=beta*tf;
 writeln('Washing of precipitate:',ts:15:3,' hours
 (',round(ts*60):3, ' min.)');
 td:=tt-tf-ts;writeln('Regeneration of filtration medium:',td:9:3,'
 hours      (',round(td*60):3,' min.)');
 ttm:=round(tf*60)+round(ts*60)+round(td*60);
 writeln('------------------------------------------------------');
 writeln('T O T A L :',tt:28:3,' ore (',ttm:3,' min.)');readkey;
END;
```

5. Results and their interpretation

Using the TURBO-PASCAL program, having user's procedures [6], the following values were obtained for the regressive equation coefficients:

$$b_0 = 0.16667;\ b_1 = 34.3434 \tag{25}$$

They served for the calculation of the filtration constants that are:

$$K = 0.05824\ m^2/h;\ C = 0.00485\ m^3/m^2 \tag{26}$$

The suspension concentration of the worn out water being considered 30 %, for a quantity of 10 tons of mud, it resulted a filtrate volume of 23.333 m^3, that is:

$$V_t = \frac{V_t}{A_{filter}} = \frac{23.333}{0.1} = 233.33 \frac{m^3}{m^2} \tag{27}$$

Effectively replacing the numerical data in the relations (16) and (17), it results:

$$A = 1.402 \cdot 10^6;\ B = 58.33 \tag{28}$$

for $\tau_a = 0.4$ h, $\beta = 0.4$ and $\gamma = 0.10$. From the equation (22) it resulted:

$$z^* = 1,497.922 \tag{29}$$

Accordingly with the relation (23), it resulted n^*=1,872 cycles.

Replacing this optimal value in relation (18), one gets:

$$\tau_t^* = 1,556\,\mathrm{h} \tag{30}$$

The time for a filtration cycle in optimal conditions is:

$$\tau_c^* = \frac{\tau_t^*}{n^*} = \frac{1,556}{1,872} = 0.831\,\mathrm{h}\ (50\ \mathrm{minutes})$$

Taking into account the previous relations it results for the filtration cycle in optimal conditions the following structure:

Proper filtration: 0.287 hours (17 minutes)

Cake washing: 0.115 hours (7 minutes)

Filtration medium regeneration: 0,429 hours (26 minutes).

Analysing this structure one notices that, though the time for the regenerating of the filtration medium exceeds the sum for the other two operations, the global filtration time is lower. That is explained because the filtration rate decreases very much for high times of filtration. Obviously, these results are not conclusive for any piece of industrial equipment; those previously depicted being only the modality for the problem treatment, which represents the experimental base for the computer program. The program can be successfully used for any filtration apparatus. We do not consider that the analysed example is conclusive, but the modality of the problem's tackling. This optimization needs an improvement, taking into account the economical aspects, using an economical optimization criterion. However, the contribution of technological optimization is great enough in the overall optimization.

6. Conclusions

a) The paper depicts a method for the optimization of discontinuous filtration using geometrical programming and setting off from experimental data.

b) A program based on the depicted algorithm that allows the obtaining of the optimal structure for a filtration cycle, using any filter that runs discontinuously has been elaborated; both the modality of optimization and the elaborated program may be used in industry and in environmental protection, thus one can contribute at the growing of the economical efficiency for the equipment's exploitation.

c) Even though, in environmental protection, we had not as an aim an economical optimization, one can quickly notice that the application of this method for the filters that are used in industrial domains can produce a considerably grow of the benefits, even if the optimization needs some experiments on the optimized filter.

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