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CALCULATION OF THE DOSE RATE IN AN EXTERNAL POINT OF A CYLINDRICAL GAMMA RADIOACTIVE SOURCE

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abstract: In this paper basic formulas are developed for the calculation of the absorbed dose rate outside of a gamma radiation source of an empty cylinder shape. The dose rate was calculated in an arbitrary point assuming a uniform distribution of the radionuclide inside the source and neglecting the attenuation build-up factor and self-absorption.

key words: particle fluency; absorbed dose; energy flux density; point of interest.

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Introduction

In dosimetry and radioprotection the determination of characteristic quantities is done either through measurement with suitable instruments or by computation. Any radioactive source is characterized by its activity (Λ) and by its constituent radionuclide through the schema factor (s). The factor s represents the number of gamma photons of a certain energy (E) and intensity emitted by disintegration [1].

Consider a radioactive monoenergetic source (which emits photons of same energy) presumed as point. It is placed in the center of a sphere of radius *r* and emits gamma rays in the environment. The gamma photons emitted are distributed evenly on the surface of the sphere. The ratio between the activity of the source and the surface area of the sphere is designated as the flow of the fluency of the gamma rays [2].

The absorbed dose rate (*D*) at a distance *r* from the source is equal with the product between the photon fluency flow and the incident photons energy. The absorbed dose rate in the case of a point monoenergetic source is given by the following equation [3]:

$$D = k_{\gamma} \frac{L}{r^2} \tag{1}$$

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Constant k_γ is a characteristic of the gamma emitting radionuclide, which forms the radioactive source. It depends on the energy of the photons emitted from the source, on the schema factor and on the photons absorption coefficient by the crossed medium.

The absorbed dose rate is the absorbed dose in unit time. An important distinction should be made: the absorbed dose refers to any type of radiation, which can produce ionization both directly and indirectly. A quantity named kerma [4] is used for the gamma rays, which ionize indirectly, that is the ratio between the kinetic energy transferred by the gamma photons to the electrons from the unit mass of medium crossed.

In the case of extended sources of gamma rays, the absorbed dose is calculated by considering the source as being made of an ensemble of point sources, which fill evenly the volume of the extended source [5]. In this paper the contribution of each source is taken into account by integrating on the whole volume.

Calculation of the dose rate in an external point of a tubular radioactive source

The radioactive source of tubular shape is formed of nuclide with total activity Λ. The cylindrical tube has the inner radius, the height h and the wall thickness $x \ll R_0$. In order to calculate in an external point of the source it is assumed that the radioactive cylinder is built of a multitude of point sources. The specific volumic activity of the source is defined by the relation:

$$\Lambda_v = \frac{\Lambda}{2\pi R_0 hx} \tag{2}$$

The quantities used in the calculus of the fluency of the gamma rays produced by the radioactive cylinder [6] are shown in Fig. 1.

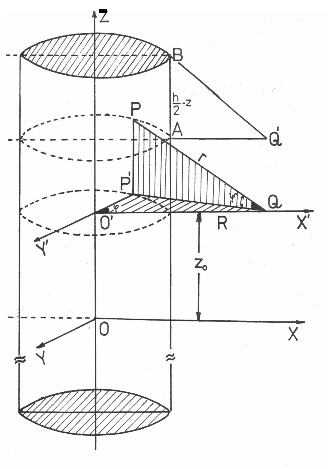

Fig. 1 *The position of the point of interest Q placed arbitrarily outside of the radioactive tubular source, where the dose rate is calculated.*

The origin was fixed at the middle of the cylinder axis. This point is placed at the intersection of the cylinder axis with the horizontal plane XOY that transects the radioactive tube at the half height.

A horizontal plane X'O'Y' parallel with plane XOY is considered, which contains the point Q, outside the source where the dose rate is calculated. An arbitrary point P inside the cylindrical source is placed at a distance z from the system of axes XOY and at a distance

$z - z_0$ from the horizontal plan, which goes through point of interest Q. The position of point Q is determined as follows: distance z_0 to the horizontal plane XOY, distance R from the axis OZ, which makes the angle φ with the axis O'Y' and distance r from the point P, considered as a point source. The projection P' of the point P on the plane X'O'Y' is found at the distance: $R^2 + R_0^2 - 2R_0R\cos\varphi$ from point Q.

The distance r between the point source P and the point of interest Q is calculated from the triangle P'PQ.

$$r^2 = R_0^2 + R^2 + (z - z_0)^2 - 2RR_0\cos\varphi \qquad (3)$$

The absorbed dose in point Q characterized by the parameters R, r, z_0, has to be evaluated. As the point where the dose is calculated is in air no absorption is taken into account [7].

Given the volume of the cylindrical radioactive source (eq. 4), the elementary volume corresponding to infinitesimal variations dφ and dz_0 will be (eq. 5).

$$V = 2\pi R_0 hx \qquad (4)$$

$$\mathrm{d}V = R_0\,\mathrm{d}\varphi \times \mathrm{d}z_0 \qquad (5)$$

The dose rate in point Q produced by the cylindrical radioactive source is:

$$D = k_\gamma \frac{\Lambda}{2\pi h} I \qquad (6)$$

where I is the integral:

$$I = \int_0^{2\pi} \int_{-\frac{h}{2}}^{+\frac{h}{2}} \frac{\mathrm{d}\varphi\,\mathrm{d}z_0}{R^2 + R_0^2 - 2RR_0\cos\varphi + (z - z_0)^2} \qquad (7)$$

By substituting $\mathrm{tg}\frac{\varphi}{2} = t$, taking into account that

$$\mathrm{d}\varphi = \frac{2\,\mathrm{d}t}{1+t^2}; \qquad \cos\varphi = \frac{1-t^2}{1+t^2} \qquad (8)$$

and after integrating with respect to the variable t, relation (7) becomes:

$$I = 2\pi \int_{-\frac{h}{2}}^{+\frac{h}{2}} \frac{\mathrm{d}z_0}{\sqrt{\left[(R+R_0)^2 + (z_0 - z)^2\right]\cdot\left[(R-R_0)^2 + (z_0 - z)^2\right]}} \qquad (9)$$

Integral (9) is transformed by another change of variable i.e. $z_0 – z = u$, into:

$$I = 2\pi \int_{-\frac{h}{2}-z}^{\frac{h}{2}-z} \frac{\mathrm{d}u}{\sqrt{\left[\left(R+R_0\right)^2+u^2\right]\cdot\left[\left(R-R_0\right)^2+u^2\right]}} \tag{10}$$

If the limiting condition $R = 0$ is imposed (point Q is situated on the axis of the cylinder) the integral becomes:

$$I = \frac{2\pi h}{R_0^2 + z^2 - \frac{h^2}{4}} \tag{11}$$

By replacing the value of the integral (11) in the formula of the dose rate (6) the later becomes:

$$D = \frac{k_\gamma}{z^2} \frac{1}{1 + \frac{R_0^2}{z^2} - \frac{h^2}{4z^2}} \tag{12}$$

The dose rate calculation outside the cylindrical source is presented next, in function of the position of the point of interest Q towards the source.

A vertical plane is drawn through point Q', placed in the same horizontal plane as point P (distant by z from the plane XOY) which contains the center of the cylinder, as well. A triangle Q'AB is formed in this plane (see Fig. 1). In this triangle the distance z is related to the angleΨ. This permits to replace z by Ψ as the vertical variable.

It is noticed that three distinct situations are met in the estimation of the dose, depending on the position of the point of interest Q on the vertical of the tube.

Case a. $z < -\frac{h}{2}$ then: $\frac{h}{2} - z > 0$ and $-\frac{h}{2} - z > 0$ that is both limits are positive.

A function change (13) should be made in order to solve integral (10):

$$u = (R - R_0)\ \mathrm{tg}\,\Psi \tag{13}$$

Using eq. (13) integral (10) becomes:

$$I = 2\pi \int_{\Psi_1}^{\Psi_2} \frac{\frac{R-R_0}{\cos^2\Psi}\mathrm{d}\Psi}{\sqrt{\left[\left(R+R_0\right)^2+\left(R-R_0\right)^2\mathrm{tg}^2\,\Psi\right]\cdot\left[\left(R-R_0\right)^2+\left(R-R_0\right)^2\mathrm{tg}^2\,\Psi\right]}} \tag{14}$$

The limits of integral (14) are:

$$\Psi_1 = \mathrm{arctg}\frac{-\frac{h}{2}-z}{\left(R-R_0\right)} \qquad \Psi_2 = \mathrm{arctg}\frac{\frac{h}{2}-z}{\left(R-R_0\right)} \tag{15}$$

The final form of integral (14) is:

$$I = \frac{2\pi}{R+R_0}\int_{\Psi_1}^{\Psi_2}\frac{\mathrm{d}\Psi}{\sqrt{1-k^2\sin^2\Psi}} \tag{16}$$

where $k^2 = \dfrac{4RR_0}{R+R_0}$.

Integral (16) designated by $F\left(\Psi,k\right)$ is the second kind elliptical function, of module k and argument Ψ. The formula of the dose rate in an external point of a gamma ray source characterized by the coordinate $z < -\dfrac{h}{2}$ is:

$$D = k_\gamma \frac{\Lambda}{h\left(R+R_0\right)}\left[F\left(\Psi_2,k\right)-F\left(\Psi_1,k\right)\right] \tag{17}$$

Case b. $z > \dfrac{h}{2}$ then $\dfrac{h}{2}-z<0$ and $-\dfrac{h}{2}-z<0$ that is both limits are negative.

In these conditions the substitution $u \to -u$ should be made. With the same change of function as in case "a" the limits of the integral (16) become:

$$\Psi_1 = \operatorname{arctg}\frac{\frac{h}{2}+z}{\left(R-R_0\right)} \qquad \Psi_2 = \operatorname{arctg}\frac{-\frac{h}{2}+z}{\left(R-R_0\right)} \tag{18}$$

By solving integral (16) and introducing it in eq. (6), the expression of the dose rate produced by the whole source in point Q at a vertical distance $z > \dfrac{h}{2}$ is obtained:

$$D = k_\gamma \frac{\Lambda}{h\left(R+R_0\right)}\left[F\left(\Psi_1,k\right)-F\left(\Psi_2,k\right)\right] \tag{19}$$

Case c. $-\dfrac{h}{2} < z < \dfrac{h}{2}$ then $-\dfrac{h}{2}-z<0$ and $\dfrac{h}{2}-z>0$.

With these conditions integral (10) is written as:

$$I = 2\pi\left[\int_{-\frac{h}{2}-z}^{0}\frac{\mathrm{d}u}{\sqrt{\left[\left(R+R_0\right)^2+u^2\right]\left[\left(R-R_0\right)^2+u^2\right]}}+\int_{0}^{\frac{h}{2}-z}\frac{\mathrm{d}u}{\sqrt{\left[\left(R+R_0\right)^2+u^2\right]\left[\left(R-R_0\right)^2+u^2\right]}}\right] \tag{20}$$

In the first integral of eq. (20) the variable change $u \to -u$ is needed again. The integral limits used in this case are:

$$\Psi_1 = \operatorname{arctg}\frac{\frac{h}{2}+z}{\left(R-R_0\right)} \qquad \Psi_2 = \operatorname{arctg}\frac{\frac{h}{2}-z}{\left(R-R_0\right)} \tag{21}$$

Solving integrals from eq. (20) with the limits of (21) leads to the following expression of the dose rate:

$$D = \mathrm{k}_\gamma \frac{\Lambda}{h\left(R+R_0\right)}\left[F\left(\Psi_1,k\right)+F\left(\Psi_2,k\right)\right] \tag{22}$$

Particular cases

1. Consider the particular situation $z = 0$ which corresponds to the positioning of point Q in the plane XOY distant R from the center of the radioactive cylinder. If $z = 0$ is considered in the expressions of the limits (21) it follows:

$$\Psi_1 = \Psi_2 = \operatorname{arctg}\frac{h}{2\left(R-R_0\right)} \tag{23}$$

Using relation (23) together with (22) the expression of the dose rate gets the following particular form:

$$D = 2k_\gamma \frac{\Lambda}{h\left(R+R_0\right)} F\left(\Psi,k\right) \tag{24}$$

2. If the point of interest Q is placed in the plane of the radioactive cylinder base, then $z = \frac{h}{2}$ and the limits (21) become:

$$\Psi_1 = \operatorname{arctg}\frac{h}{R-R_0} \qquad \Psi_2 = 0 \tag{25}$$

With the limits (25) the dose rate is written:

$$D = k_\gamma \frac{\Lambda}{h\left(R+R_0\right)} \int_0^{\operatorname{arctg}\frac{h}{(R-R_0)}} \frac{\mathrm{d}\Psi}{\sqrt{1-k^2\sin^2\Psi}} = k_\gamma \frac{\Lambda}{h\left(R+R_0\right)} F\left(\Psi,k\right) \tag{26}$$

Formula (26) can be obtained directly by using the geometry concerning the tubular source shown in Fig. 2.

A photon emitted from a certain point P of the wall of the source of coordinates $R_0\cos\varphi$, $R_0\sin\varphi$ and z gets in the point of interest Q situated in the plane of the base of the cylinder, outside of it, wandering through distance r. The point Q is found at distance R from the axis of the cylinder.

The distance r is calculated from the triangle QPP' situated in a vertical plane:

$$r^2 = R_0^2 + R^2 + z^2 - 2RR_0 \cos\varphi \tag{27}$$

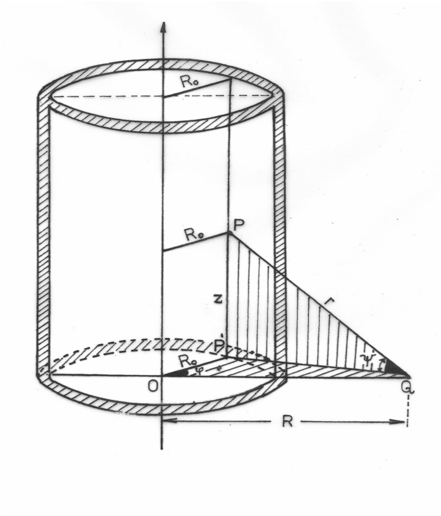

Fig. 2 *The definition of the position of the point Q placed in the plane of the base of the radioactive cylinder, outside of it.*

The volume element expressed in function of the variables φ and z is given by relation (5) while the elementary volumetric activity by eq. (2).

The dose rate in point Q produced by the whole source is:

$$D = 2R_0 \times \Lambda_\nu k_\gamma \int_0^\pi \int_0^h \frac{\mathrm{d}\varphi\,\mathrm{d}z}{R_0^2 + R^2 + z^2 - 2R_0 R\cos\varphi} \tag{28}$$

The integral (28) with respect to the angle φ is solved by using substitutions (8). It is followed by integration with respect to the variable z, the distance from a certain point to the base of the cylinder.

$$I = \pi \int_0^z \frac{\mathrm{d}z}{\sqrt{\left[z^2 + (R-R_0)^2\right]\cdot\left[z^2 + (R+R_0)^2\right]}} \tag{29}$$

In formula (29) the variable z is replaced by Ψ, the angle between the straight line r with the horizontal:

$$z = (R - R_0)\,\mathrm{tg}\,\psi \tag{30}$$

Solving the integral (29) yields successively the following forms:

$$I = \frac{\pi}{R+R_0} \int_0^{\mathrm{arctg}\frac{h}{(R-R_0)}} \frac{\mathrm{d}\Psi}{\sqrt{1-k^2\sin^2\Psi}} = \frac{\pi}{R+R_0} F(\Psi, k) \tag{31}$$

where k^2 and $F(\Psi, k)$ were defined above *I*.

The integral (31) is introduced in the expression of the dose rate (6):

$$D = k_\gamma \frac{\Lambda}{h(R_0 + R)} F(\Psi, k) \tag{32}$$

It is noticed that an expression identical with eq. (26) is obtained:

As the angle Ψ depends on *z*, the formula of the dose rate for two values of z is:

$$D = k_\gamma \frac{\Lambda}{h(R_0 + R)} \left[F(\Psi_1, k) - F(\Psi_2, k) \right] \tag{33}$$

where $\Psi_1 = \operatorname{arctg} \frac{z}{|R - R_0|}$ $\Psi_2 = \operatorname{arctg} \frac{h - z}{|R - R_0|}$ (34)

3. If point Q is inside, on the empty cylinder axis, then $R = 0$, $k = 0$.

The dose rate expressed by eq. (17) gets the following form:

$$D = k_\gamma \frac{\Lambda}{R_0 h} (\Psi_2 - \Psi_1) \tag{35}$$

The same result is obtained if the previous conditions are imposed in eq. (33).

It may be seen that eq. (35) for the calculation of the dose is identical with the one found directly for the same source, the point of interest being disposed on the cylinder axis [8].

Conclusions

The present theoretical treatment for the calculation of the dose rate outside of a radioactive tubular source is general because the point of interest is disposed arbitrarily in space. The equation of the dose rate was deduced with the hypothesis that the radioactive substance is distributed evenly in the source. Self-absorption as well as the absorption of radiations by the air from outside the cylinder has been neglected. The self-absorption must be taken into account in the case of bulky sources, in particular when the dimensions of the source are of same order of magnitude as the average distance covered by photons in the radioactive material.

For tubular sources with well-defined dimensions, the variable parameters are the horizontal distance to the axis of the cylinder and the angle, which determines the position of the point on the vertical.

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